Factor Relationship Diagrams:
A Tool for Planning and Evaluating Experimental Designs

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Objectives

- Motivate the Need
- Establish the Basics
- Define the Concepts
- Illustrate the Method

Why Experiment?

- To create an effect
- To prioritize or screen out a subset of factors that cause the effect
- To understand the magnitude of the impact of the factors on the effect
- To understand how the factors interact with one other in producing the effect
- **To determine the ability to achieve a sustainable effect over time and condition**

A Fundamental Model

$$Y = f(C_1, C_2, \ldots \mid N_1, N_2, \ldots)$$

Where:

Y represents a critical output or measured response of the process

C's represent causal factors that impact the value and amount of variation seen in Y

N's represents the conditions, noise variables over which the cause and effect relationships can be expected to hold.

Fundamental Purpose of DOE

- To manipulate product or process factors (X's) at specifically defined levels to evaluate their impact on one or more Y's.
- Design of experiments (DOEs) are useful in understanding cause and effect relationships between factors and responses.

What is DOE's role in understanding variables not included as design factors?

What is the role of sampling, control charting, and component of variance studies?

Thinking about the "Noise"

Noise variables in a DOE, often referred to as extraneous variables, are those x's (C's and N's) that are NOT explicitly manipulated as factors in the experiment.

How are "noise" variables often incorporated into a DOE?

Option 1: Hold them constant.

Option 2: Treat them as factors.

Option 3: Allow them to vary as "normal" and estimate the magnitude of their impact on Y.

Others?

Four Common Mistakes in Planning and Evaluating DOEs

1. "Rounding up the usual suspects"
2. Proving the obvious
3. Optimization of a small set of factors while minimizing the effects of other influential variables
4. Not evaluating the risk of extrapolation of DOE results prior to setting operational parameters

Common Reasons for Not Using DOEs in Industry

- Too resource-consuming
  - Time
  - $
  - Product
- "Tried them in the past and they have failed"
- Left confined within technical -- a part of the statistical toolbox and used by engineering and statisticians

A DOE Approval Request Form

Date: 2/21/xxxx **Product:** MPA-52C4

Submitter: J. Evans **Process(es):**

Expected Start: 3/7/xxxx **Expected Completion:** 3/11/xxxx

Purpose/Objective:
To reduce the variation in gloss and color between batches.
To screen the effects of factors believed to cause differences between paint batches primarily with respect to viscosity and color (Lab).

Brief outline of the experiment design to be used:
A 6-factor, resolution IV fractional factorial

Experiment parameters:

	Factors	Levels	Predicted Effects
1.	Mix Time (TI)	5 (-) 15(+)	+ preferable
2.	Blade Speed (BS)	40 (-) 65 (+)	strong interaction with mix time
3.	Mix Temp (TE)	70 (-) 350 (+)	little effect
4.	Resin Type (R)	T02 65 C (-) T02 173 C(+)	- preferable

Response Variables: Analytic gloss and color readings

Description of preceding and expected subsequent events:
Based on this DOE, we expect to be able to verify choice of resin type and to improve processing based on the expected mix time by blade speed interaction. A follow-up experiment (requiring a reduced number of production batches) is to be run for validation and optimization purposes.

Expected resources required:
Eight containers of T02 173. 16 production batches. Overtime hours (between 8 and 16) in spray out and QC lab. Additional oven from acrylics lab to be used to speed up drying time.

Management Approval(s) ______________________

Date: ____________

One Possible Design Table

TI	BS	TE	R	L,a,b	Gloss
-	-	-	-	?	?
-	-	-	+	?	?
-	-	+	-	?	?
-	-	+	+	?	?
-	+	-	-	?	?
-	+	-	+	?	?
-	+	+	-	?	?
-	+	+	+	?	?
+	-	-	-	?	?
+	-	-	+	?	?
+	-	+	-	?	?
+	-	+	+	?	?
+	+	-	-	?	?
+	+	-	+	?	?
+	+	+	-	?	?
+	+	+	+	?	?

Some Basic Procedural Information

Assume appropriate work has led to the planned DOE. *What might this be?*

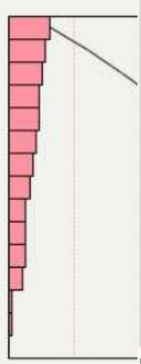
The experiment is to be completely randomized and one sample of paint to be taken from each batch made consecutively over time.

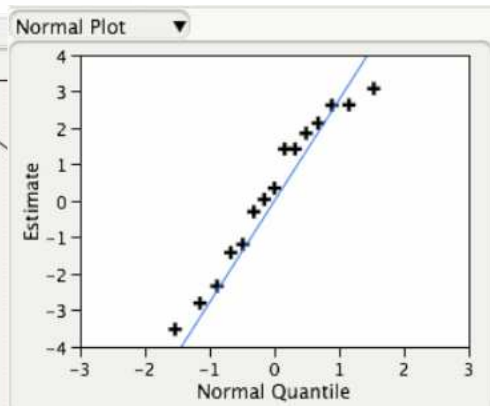
One panel will be "drawn down" in the lab and one measurement is to be taken on each panel for both color and gloss.

One Possible Outcome

Pareto Plot of Estimates

Term	Estimate
TI*BS*R	-3.521875
TE	3.081875
TI*BS*TE	-2.805625
BS*TE	2.634375
TI*TE	2.624375
BS*TE*R	-2.335625
TI*R	2.133125
TE*R	1.854375
BS*R	1.425625
BS	1.425625
R	-1.416875
TI	-1.191875
TI*BS	0.350625
TI*TE*R	-0.295625
TI*BS*TE*R	0.041875





Blue line is Lenth's PSE, from the estimates population.

What factors appear to be important in achieving and managing Y?

Why might none of the factors appear to be "statistically significant"?

Some Definitions

Design factors are those process or product parameters deliberately manipulated during the DOE.

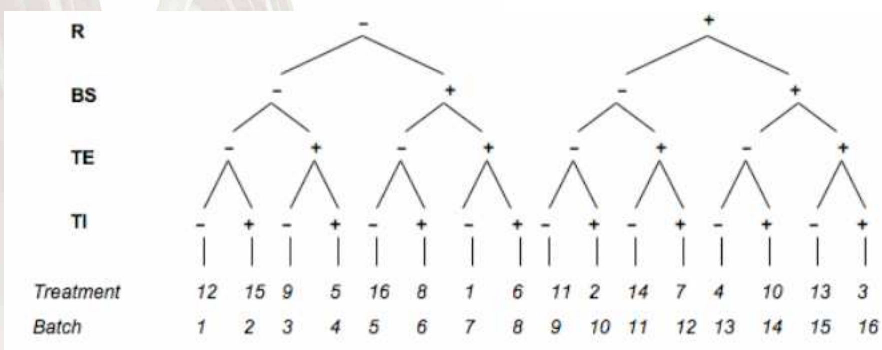
The unit structure is determined by how the noise and un-manipulated sources of variation are managed during the DOE. It consists of experimental units and materials, uncontrollable variables, and sources of variation held constant during the DOE.

The inference space represents the range of conditions over which the experimental results can reasonably be expected to repeat.

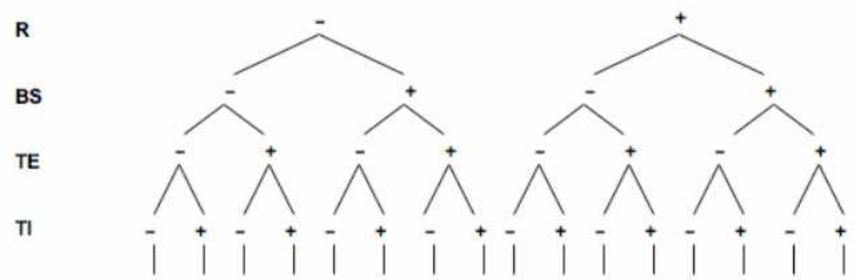
The Factor Relationship Diagram

(FRD)

The FRD is a graphical, schematic tool that portrays the relationships between the potential sources of variation in a DOE -- the design factors and the unit structure.



FRD Showing Sources of Variation Changing Between DOE Treatments



Treatment	12	15	9	5	16	8	1	6	11	2	14	7	4	10	13	3
Batch	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Within Batch	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Panel	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Measurement	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Some Questions About the Unit Structure

How many batches will be made during this DOE?

How will inconsistencies within a batch impact DOE results?

Where will the impact of setup factors, not included as design factors, show up in the data from the DOE?

What about measurement error?

What sources represent “experimental error”?

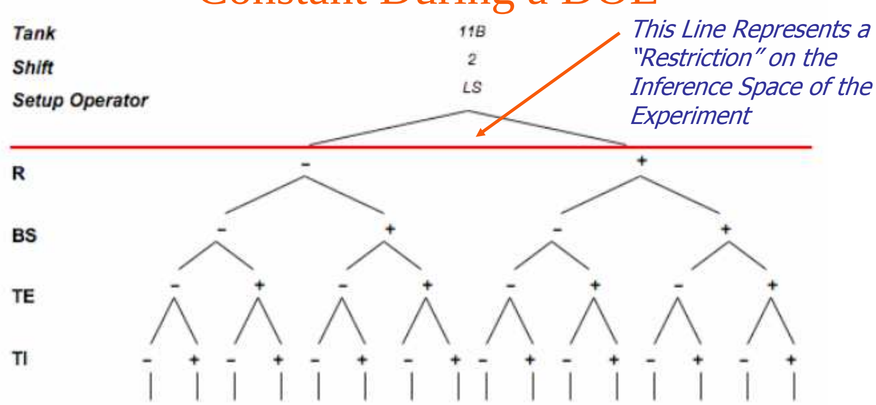
Considering the Risk of Extrapolating DOE Results

- In addition to those elements of the unit structure changing between treatment combinations, there are additional sources of variation held constant during a DOE.
- These are considered to be "restrictions" on the the experimental inference space.

Examples:

1. Multiple mixing tanks are used in the batch process. However, only one tank was allocated for use during the DOE.
2. All batches to be made during the DOE are to be during 2nd shift. Hence, the same operator will set up each batch.

The FRD: Sources of Variation Held Constant During a DOE



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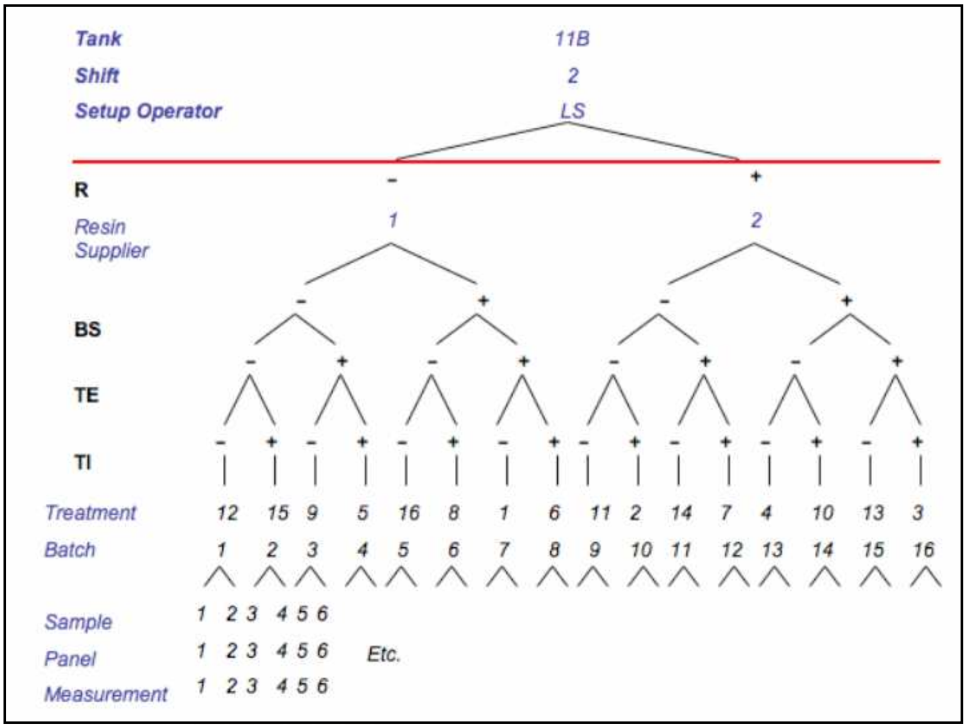
Measurement	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Some More Questions About the Unit Structure

1. When is it appropriate to hold sources of variation constant during a DOE?
2. What is the impact of these sources that are held constant on our ability to extrapolate the DOE results into process, product, and future studies?
3. What is the impact of the restricted inference space on our evaluation and interpretation of "statistical significance"?

Unit Structure Complications

- The 16 treatment combinations are to be completely randomized.
- However, the eight batches of paint made with resin-type 65C (-) are made with a resin from a different supplier from resin type 173C (+).
- Additionally, the engineer wants to obtain two samples from each batch of paint. Each sample will be used to draw down a separate panel. Hence, two measurements are obtained for each treatment combination.



Insightful Questions

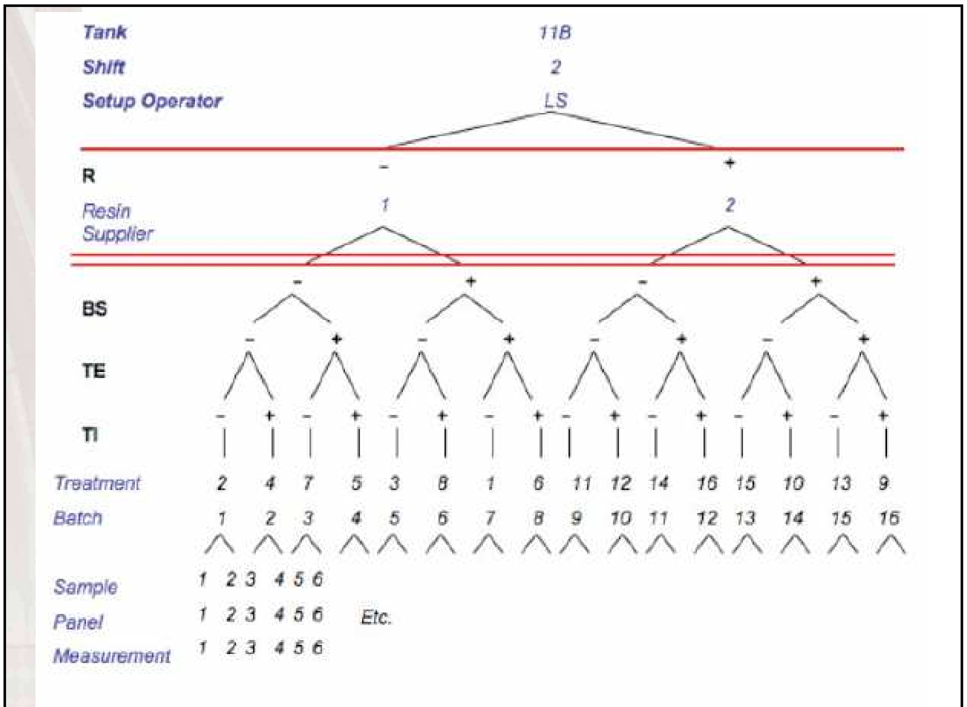
- How many batches of paint are needed?
- What additional resources will be required?
- What additional information is available? What can we learn about from the data?
- What is the impact of the additional unit structure considerations on the interpretation of DOE results?

Restrictions on Randomization

The FRD contains information on the design factors, unit structure, and any lines of restriction.

Lines of Restriction exist when:

- *Sources of variation in the unit structure are held constant*
- *A potential source of variation is not randomly allocated across the treatment combinations.*
- *Treatments combinations are not run in random order over time.*



Three Key Insights

1. The estimated effect of a factor is equal to the true effect of that factor + the effect of any unit structure confounded with the change in the levels of that factor + the effect of noise!

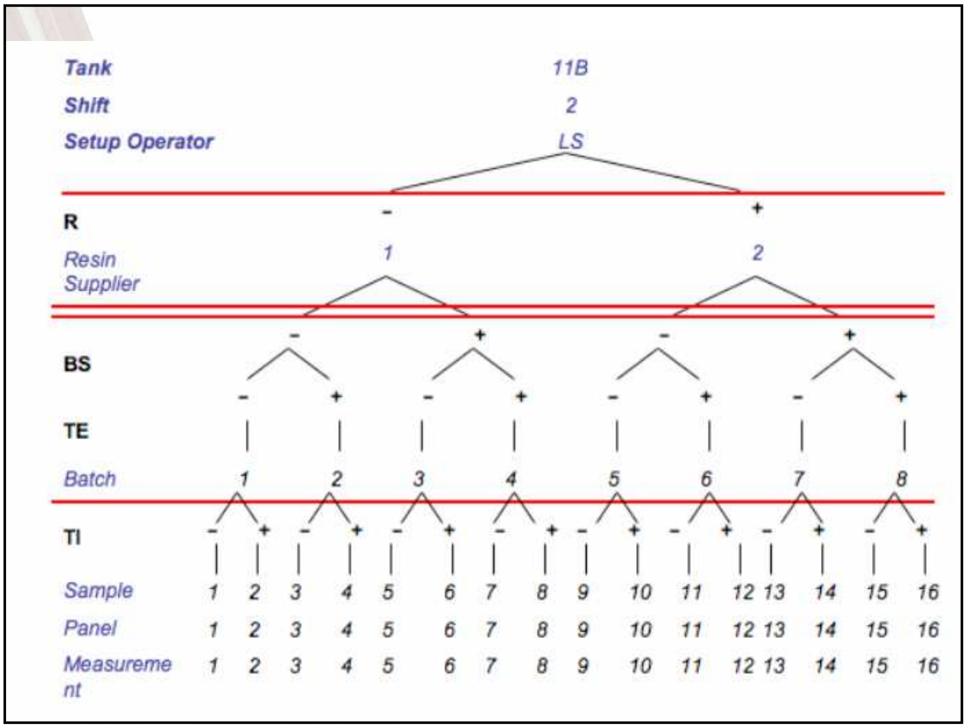
$\hat{R}$ **= True R Effect + Supplier Effect + "Noise"**

2. It is not "fair" to compare the effect of Resin Type in the DOE to the effect of the other factors. *Why?*
3. The ability to understand the impact of restricting randomization, either with respect to a factor or with regards to the allocation of materials, is completely dependent upon one's knowledge about the unit structure.

Many, Many, Many Choices!

- Fractional Factorial with more factors
- Taguchi design including a noise variable as a factor.
- Restricting on Mix Time -- only make 8 batches instead of 16.

And so on.



Evaluating Proposed DOEs

1. There are more options with regards to how one chooses to manage the unit structure than there are choices of design structures (assuming balanced and orthogonal).
2. DOEs cannot be effective when used outside of a sequential process of observing, questioning, sampling and experimenting!
3. "Statistical significance" cannot be the goal. The designer of the DOE determines the possibility of significance based on the management of the unit structure.

Some Key Insights

1. It is not validation of current knowledge that leads to improvement. An experiment must provide the opportunity for engineering theory and experimental results to fail to agree.
2. There is never a "perfect" experimental plan. If the design is planned according to current knowledge and predictions, then knowledge will result.
3. No experiment is a failed experiment if used to gain insight into the ability to use factors to achieve repeatability of effects.

Some Key Insights

4. Restrictions on randomization are only dangerous when their implications on the usefulness of the results is not considered prior to running the DOE.
5. One cannot make conclusions from DOE results without understanding the contributions from the unit structure!
6. Develop multiple DOE plans and compare them with respect to resources required and potential risk of loss of information before running!

A Quote by Dr. W Edwards Deming

"Unfortunately, future experiments (future trials, tomorrow's production) will be affected by environmental conditions different from those that affect this experiment.

It is only by knowledge of the subject matter, possibly aided by further experiments, to cover a wider range of conditions, that one may decide, with a risk of being wrong, whether the environmental conditions of the future will be near enough the same as those of today to permit the use of results in hand."

References

- Bergerud, Wendy A., "Displaying Factor Relationships in Experiments", The American Statistician, Vol. 50, No. 3, August, 1996.
- Hild, Cheryl and Doug Sanders, "Factor Relationship Diagrams: A Tool for Experimenters", Wiley's Encyclopedia of Reliability and Statistics, 2008.
- Sanders, Doug and J. Coleman, "Recognition and Importance of Restrictions on Randomization in Industrial Experimentation",Quality Engineering, Vol. 15, No. 4, 2003.